

MODELLING OF INTERNAL STRESSES IN GRINDING CHARGES

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Summary. Physically realistic methods are a necessity to close the gap between reality and numerical result in modelling of tumbling mills. A problem is that tumbling mills often operate in a metastable state because of the difficulty to balance the rate of replenishment of large ore particles from the feed with the consumption in the charge. Understanding of the charge motion within the mill is of significance in mill optimisation. Both the breakage of ore particles and the wear of liners/ball media are closely linked to the charge motion. In this work, a ball charge and its interaction with the mill structure is modelled with the smoothed particle hydrodynamic (SPH) method. The mesh free formulation and the adaptive nature of the SPH method result in a method that handles extremely large deformations and thereby suits modelling of grinding charges and pulp liquids. The flexible rubber lifter and the lining are modelled with the finite element method (FEM). A hyper-elastic model governs the elastic behaviour of the rubber.

The comminution process is complex and to include all phenomena that occur in a single numerical model is today not possible. Therefore, modelling the physical interaction between charge, mill structure and pulp liquid is the major goal in this work. The SPH-FEM model can predict responses of the mill structure e.g. stress and strain. All parts of the mill system will affect its response and the model gives the opportunity to study the influence of the mill structure and e.g. pressure and shear stresses in the charge. This computational model also makes it possible to predict, the contact forces for varying mill dimensions, liner combinations and pulp densities. By comparing numerical results with experimental measurement from grinding in a pilot mill equipped with an instrumented rubber lifter a validation is done. The deflection profile of the lifters obtained from SPH-FEM simulation shows a reasonably good correspondence to pilot mill measurements as measured by an embedded strain gauge sensor. This model gives information on the grinding process in tumbling mills and better correlation with experimental measurements.

1 INTRODUCTION

Grinding of material in tumbling mills is a highly important size reduction process for the mining industry. The process is difficult to control and energy inefficient. The grinding process in a tumbling mill is strongly connected to the charge properties, liner design, rotational speed, filling rate, pulp fluid properties etc. Both the breakage of ore particles and the wear of liners/ball media are closely linked to the charge motion. To study these phenomena in a physically correct manner, suitable numerical models for different parts of the mill system has to be utilised. Validation of these models is of major importance.

A traditional approach to study the charge behaviour is to use the discrete element methods (DEM). The method was introduced by Cundall [1] for analyses of rock mechanic problems. When applied to comminution it gives an opportunity to study several aspects of grinding in detail than has been possible to date, e.g. charge viscosity and charge size distribution, collision forces, energy loss spectra and power consumption. An initial attempt to use DEM to describe the interaction of large grinding balls and the lining was presented by Rajamani [2]. However, some improvements of today's DEM models can be identified; for example, charge normal and shear stress can not be found with rigid DEM particles. Jonsén et al. [3] took one step towards a more physically realistic mill model. They used a combined DEM-FEM model to study the interaction between the charge and the mill structure.

For astrophysical problems in open space, the smoothed particle hydrodynamics (SPH) method was invented, see [4,5]. It is a mesh free, point based method for modelling fluid flows, which has been extended to solve problems with material strength. Today, the SPH method is used in many areas such as fluid mechanics (for example; free surface flow, incompressible flow and compressible flow), solid mechanics (for example; high velocity impact and penetration problems) and high explosive detonation over and under water. In the SPH method, a problem domain is represented by a set of particles or points, see [6]. Besides representing the problem domain, the points also act as the computational frame for the field approximation. Each point is given a mass and carries information about spatial coordinate, velocity, density and internal energy. Other quantities as stresses and strains are derived from constitutive relations. The mesh free Lagrangian formulation and the adaptive nature of the SPH method result in a method that handles extremely large deformations. In this work, a step towards a physically correct description of the ball charge is taken by using SPH to model the ball charge.

One of the most developed numerical methods is the finite element method (FEM). FEM is a numerical solution method based on continuum mechanics modelling, a constitutive relation for the actual material is described and the governing equations are solved [7]. Varieties of different constitutive models for a large number of materials are implemented in modern finite element (FE) code. A material model approximates a real physical behaviour. The comminution process is complex and to include all phenomena that occur in a single numerical model is today not possible. Therefore, modelling the physical interaction between the charge and the mill structure is important if the internal stress in the charge is studied. The SPH-FEM model can predict the classical DEM results, but can also predict responses from

the mill structure like, e.g., stress and strain, see [8]. All parts of the mill system will affect its response and a SPH-FEM model gives the opportunity to study the pressure and shear stresses in the charge. In this work the internal stresses in the charge is studied, as it may give a better understanding of the grinding process. The validation of this task is done by comparing numerical results with experimental measurements from grinding in a pilot mill equipped with an instrumented rubber lifter.

2 EXPERIMENTAL MEASUREMENTS

Pilot mill experimental measurements have been done to study the interaction between the charge and lining. As the mill is rotated the charge will load and deform the lining. In the measurements, the deflection of a rubber lifter captured as it moves through the charge.

2.1 Measurement system

One lifter is equipped with a sensor and a simplified view of the sensor is shown in Fig. 1. The mill has several lifters and the one equipped with a leaf spring (marked 1) is whose deflection is measured by the strain gauge (marked 2). As the mill rotates and the lifter with the sensor dips into the charge, the force acting on the lifter increases, which in turn, causes a deflection. The strain gauge mounted on the leaf spring converts this deflection to an electric signal. This signal is amplified, filtered and transmitted to a computer. Metso Minerals have marketed the sensor system under the name Continuous Charge Measurement system (CCM), see [9]. A typical deflection profile of the sensor signal and an attempt to divide it into different segments is shown in Fig. 2. The boundaries and size of the partitions are determined by engineering knowledge of the grinding process. Each segment in Fig. 2 illustrates an important dynamic event during the passage of the sensor-equipped lifter bar under the mill charge. The ordinate in Fig. 2 shows the deflection of the lifter bar, which indirectly corresponds to the force acting on it and the abscissa is the mill rotation angle with a resolution of 1° .

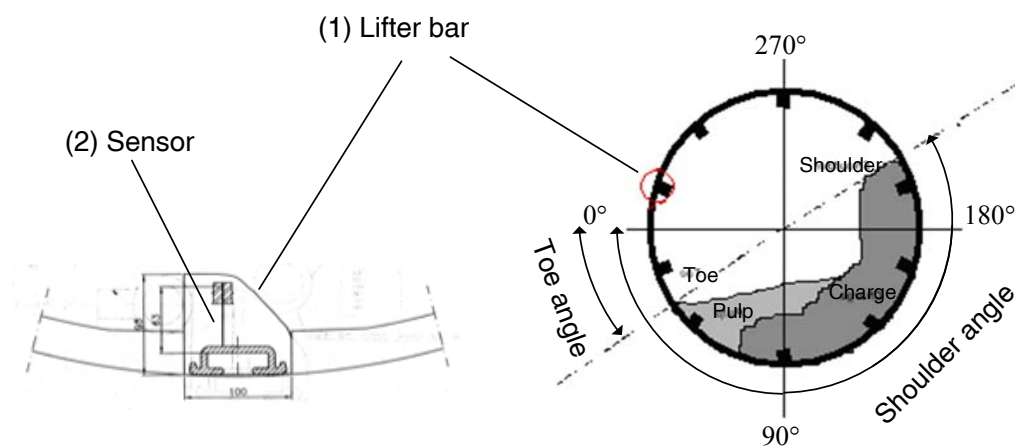


Figure 1. To the right a cross section of a mill with a horizontal reference line, the left part shows the lifter bar (1) with a strain gauge sensor embedded (2), from Tano (2005).

The sensor signature reflects different charge features such as mill volume, position and behaviour of the mill charge. Both toe region (S2) and shoulder region (S6) are well known, and can be used to calculate the volumetric mill load and the angle of repose based on the CSM-signal that is calibrated against the measured charge volume when the charge is at rest. Collectively, these data give a good measure of the location of the charge. The other segments are less known but are expected to provide information about grinding efficiency. Such features can be extracted from the sensor signal for the purpose of process monitoring and diagnosis of process performance. Table 1 provides a summary of the stages during one mill revolution. The lifter bar angles given in Table 1 correspond to the positions marked in Fig. 2, as a reference in these measurements is the horizontal line, which corresponds to 0 degrees at the 9 o'clock position. In this work, an attempt is made to identify the corresponding segments in a predicted deflection profile obtained from SPH-FEM simulation. The toe/shoulder position in particular will be inspected for validation purposes.

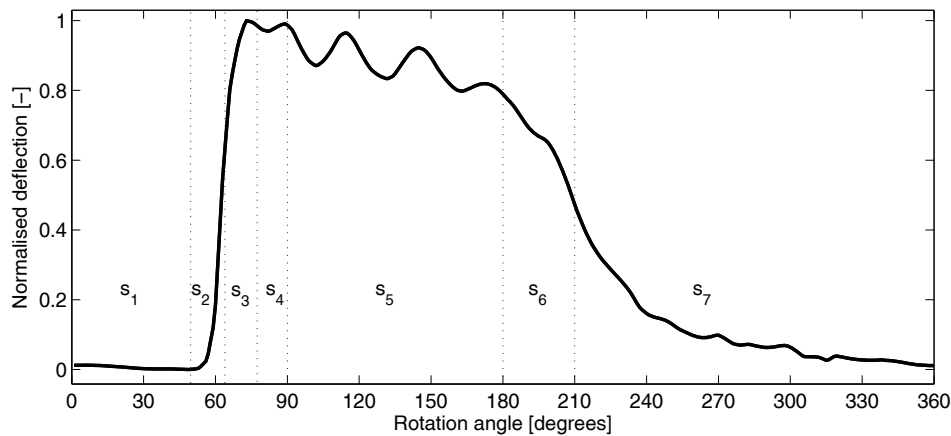


Figure 2. Segmentation of a typical sensor signal during its passage in the charge Tano [10].

Table 1. Sensor lifter bar signal segmentation and grinding load features Tano [10].

Segment	Lifter bar angle	Process feature
S ₁ : the sensor lifter bar (SL) is still in the air	< 50°	
S ₂ : the SL hits the ball charge and starts to get submerged in the charge	50°- 70°	Indicate the toe position of the charge, and if present, the slurry pool
S ₃ : the SL starts to bend forward due to turbulence in the toe area	70°- 85°	Rate of charge varies with mill speed
S ₄ : the SL is at peak bending	75°- 90°	Both speed and charge level has an influence, wear of lifter
S ₅ : the SL is moving through the charge	80°- 190°	Indication of every lifter bar hitting the charge
S ₆ : the SL has gradually decrease the bending and is at take-off position	190°- 215°	Indicates the shoulder position of the charge
S ₇ : the SL is leaving the ball charge and starts slowly to rise to an upright position	> 215°	

2.2 Experimental conditions

The pilot mill is 1.41 m in diameter and 1.22 m in length. It is a grate-discharge mill, equipped with 12 rubber lifters of square size 0.1 m and a face angle of 45 degrees. Steel balls with a diameter ranging between 10-30 mm and a density of 7800 kg/m³ were used in the experiments. The test material, a hematite pellet feed with d_{50} around 35µm and a solids density of 5200 kg/m³, was chosen to get stable grinding conditions with respect to feed size variations. Feed rate was kept constant at approx. 1.5 tonne/h. Four experiments were run with the mill speed at 73% and 78% of critical speed (n_c) for two levels of mill filling ($J = 25\%$ and 35% by volume). The embedded strain gauge sensor measured the load position (toe and shoulder) using the CCM algorithm, proprietary of Metso Minerals. More details regarding the experimental measurements can be found in [10].

3 MODELLING

To virtually reproduce a tumbling ball mill process a combined three dimensional SPH-FEM model is used. The commercial nonlinear finite element code LS-Dyna v971 R5.1 [11] has been used for the modelling and simulation of the mill. The mill structure consists of lifters, liners made of rubber, and a mantel made of solid steel. Here the rubber is modelled as a hyper-elastic material and the mill mantel is modelled as a rigid material.

3.1 SPH Charge model

The problem domain in SPH is represented by a set of particles or points. Besides representing the problem domain, the points also act as the computational frame for the field approximation. Initially, each point is given mass and coordinate information. During calculation, information about spatial coordinate, velocity, density and internal energy is stored in the each point. From constitutive relations, stresses and strains are derived. The SPH method is an adaptive Lagrangian method, which means that in every time step the field function approximations are performed based on the current local set of distributed points. Another difference from the FE method is that the points are free to move in action of the internal and external forces, there are no direct connections between them like the mesh in FE method.

The SPH method is originally a continuum-based method used to model fluids, granular and solid material. For that kind of processes the smoothing length can vary. Here, the balls are modelled in three different sizes 10, 20 and 30 mm. Each sphere size is given a mass m and radius h that correspond to the density of steel 7800 kg/m³. To reproduce the charge behaviour the smoothing function is set to a constant value. In the present case, the ball charge is modelled with 6000 SPH particles. In 3D, a sphere represents the SPH particle with its radius controlled by the value of h . To model the steel balls in the charge the sphere is given a constant mass and radius. A constitutive relation eq. 1 given by the Drucker-Prager model governs the interaction between the balls.

$$f_{dp}(p, J_2) = \sqrt{2J_2} + kp + d = 0 \quad (1)$$

Where, p is the mean pressure, J_2 , the second invariant of the deviator stress tensor and k and d are material parameters representing internal friction and cohesion, see [11]. In this work, the slope of the internal friction k has been predicted from particle-particle friction recommended by Rajamani [2]. For simplicity, the elastic properties are set constant for the actual range of density. When the SPH particles interact with each other or the lining values of mechanical properties like stresses can be calculated. The SPH particle will give a constant value of these properties within the particle.

3.2 Mill structure model

A 0.10 m slice of the mill is modelled and the diameter is 1.414 m. All structural parts of the model: rubber liners, rubber lifters and mantel are modelled with eight node solid elements. This type of element is fully integrated with a reduced integration of the pressure part to avoid volumetric locking of the element. The Blatz-Ko hyper-elastic model governs the elastic behaviour of the rubber, see [12]. Hyper-elastic models are a classic approach to model rubber material as they are considered ideally elastic. Here, the stress-strain relationship derives from a strain energy density function. The supplier of the lining provided experimental data for the rubber. In Fig 3. the SPH-FEM mill model is shown.

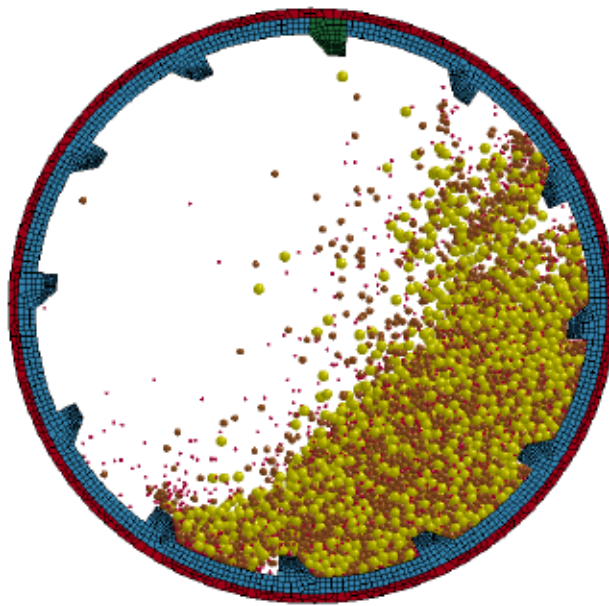


Figure 3. The 3D SPH-FEM mill model containing a graded charge, lifter and liners in rubber and a solid mantel modelled with rigid material

4 RESULTS

The mill process is simulated for four revolutions that is around 10 s in real time. The charge filling was $J = 25\%$ and speed 73% of critical rotational speed. In Fig. 4, a snapshot of the total velocity vectors and the charge movement at steady state is shown. Note, the internal kidney-shaped rotational motion of the charge, where the grinding balls merely are holding

their places and rotating in their positions. As the mill rotates charge motion is induced. During this motion particle-particle and particle-structure interactions occurs in the mill system. The contact between particles and structure of the mill results in a load to the structure of the mill. The structure will respond with deformation upon the incoming load from the charge. These deformations will give rise to strains and stresses which are dependent on the material properties of the structure. This load also induces mechanical waves in the whole structure and charge. Jonsén et al. [3] did show that a DEM-FEM model could predict the mechanical waves that travel in the mill structure. The SPH-FEM model can also show that mechanical waves travel in the charge. In Fig 5, the pressure distribution in the charge is shown for three different occasions within 30° including submerging, fully loaded and just before the next lifter will submerge. As the lifter submerges into the charge (position a), a pressure builds up in front of the lifter. In position b, the pressure is fully developed. Position c shows that the pressure follow the lifter as it moves in the charge. The collision that appears as the lifter submerges into the charge starts a mechanical wave that travels through the charge. As the mechanical wave travel through the system, the charge is compressed and unloaded several times. These waves creates fluctuations in the lifter load that is shown in the experimental study, see Fig 2. The highest pressure is found as the lifter goes into the charge.

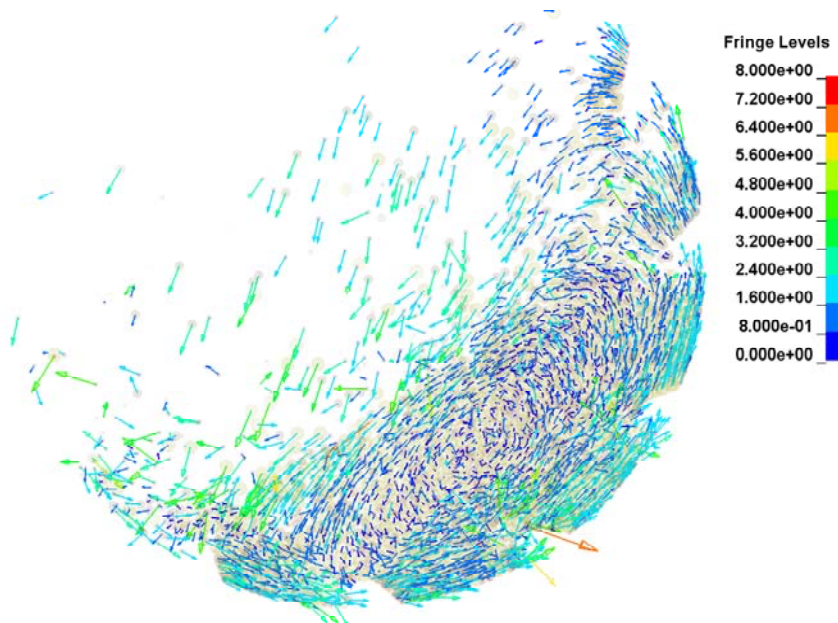


Figure 4. A snapshot of the total velocity vectors in the charge.

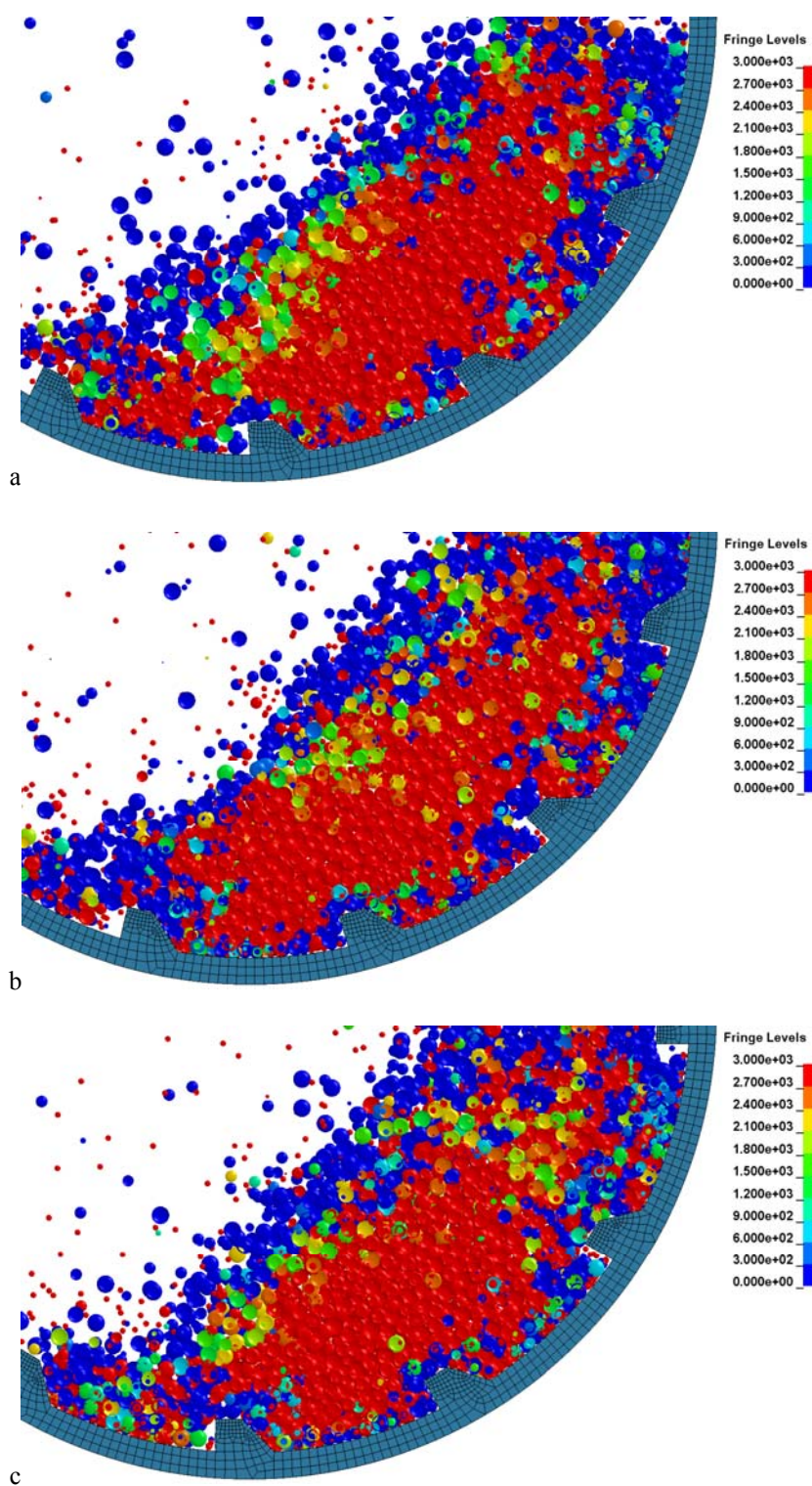


Figure 5. The pressure distribution in the charge as the lifter travels trough the charge. a) As a lifter submerges into the charge, hydrostatic pressure build up and a mechanical wave is induced and travels through the charge. b) A fully developed pressure region is found in front of the lifter. c) The pressure region moves with the lifter and a subsequent lifter is about to submerge into the charge.

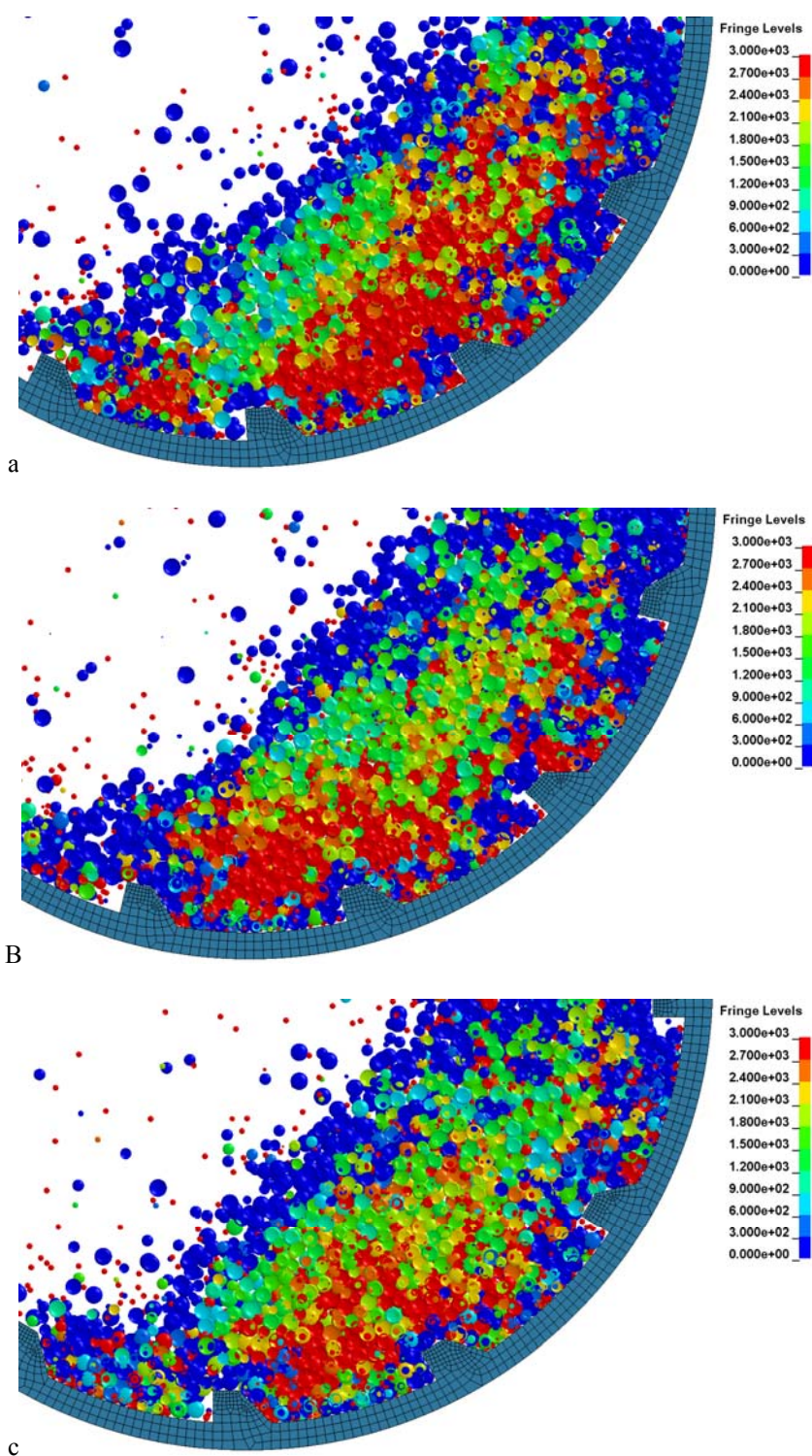


Figure 6. The maximum shear stress distribution in the charge as the lifter travels through the charge. a) As a lifter submerges into the charge, shear stress build up and a mechanical wave is induced and travels through the charge. b) A fully developed shear stress region is found in front of the lifter. c) The shear stress region moves with the lifter and a subsequent lifter is about to submerge into the charge.

At identical positions the maximum shear stress distribution for the mill is shown in Fig 6. As the lifter submerges into the charge (position a), a shear stress builds up in front of the lifter. In position b, the shear stress is fully developed. Position c shows that the shear stress follow the lifter as it moves in the charge. The magnitude and volume affected by the maximum shear stress seems to be smaller than for the pressure. The highest value for the shear stress is found in front of the lifter close to where the horizontal velocity changes direction. As for the pressure, the shear stress is also changing from low to high level as the mechanical waves travel in the system.

The deflection of the lifter during its fourth passage for the simulated and the experimentally measured is shown in Fig. 7. There are differences in the signature between measured and simulated results that probably are due to the lack of a dampening pulp liquid in the simulated charge model. What is important here is that both simulations show the same 30° separation between the peaks as the measured signal. This means that the mechanical waves are correctly predicted. The maximum peak values for measured and the simulated signature occurs around 105° . The charge toe angle is 69° for the measured and 65° for the model. The shoulder angle for measured is 202° and 190° for the model charge. The peaks are higher for the model.

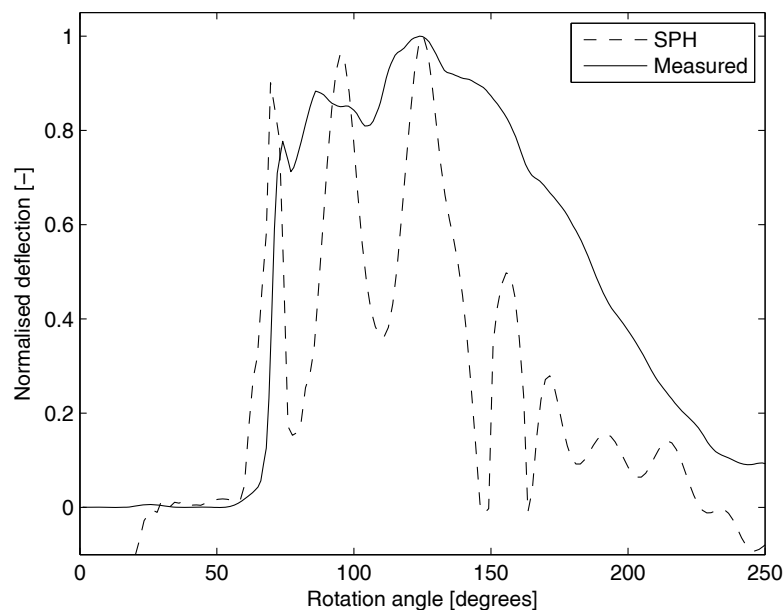


Figure 7. Displacement of the lifter during the fourth passage, solid line represents the response from the graded charge and dashed the SPH-FEM model

5 DISCUSSION AND CONCLUSION

Through a SPH-FEM model, structural response and its influence on the charge motion in a mill can be studied in great detail. Constitutive models govern the behaviour of materials in the mill. The models are calibrated with data from experimental tests or taken from literature. The SPH-FEM model gives not only the opportunity to optimize the material selection of the mill structure but also to study the internal workings of the charge. Critical response values e.g., stress and strain can be identified during the milling process. Forces and mechanical waves in the structure and charge can be found. An attractive result is the pressure and shear stress distributions and their change with the position of the lifters, see Figs 5 and 6. This result gives a better insight to the grinding process and the travelling of the mechanical waves within the charge and mill lining. Within a grinding ball, the stress is distributed. To resolve that stress for each grinding ball in a mill is possible but very computationally costly and today not reasonable. Each ball has to contain a number of FE elements to resolve the stress field. To model grinding balls as single SPH particles give a constant stress. This means that the single SPH particle provides the mean value of the stress. The peak values will not be found with this approach but it will give an idea of how the global stress field looks like.

A combination of Figs. 4 to 6 reveals that the area with slow-moving grinding balls overlaps the areas with maximal internal charge pressure and shear stress. This is an important result, since this combination may be considered to maximise the attrition process that generates the fine particles in grinding. Being able to predict the extension of this area, and the magnitude of the stresses in it, opens up the possibility for improving the energy efficiency in tumbling mills.

The presented SPH-FEM model gives a physically accurate presentation of the system. The liner design has a large influence on the charge motion but the charge is also changed during its passage through the mill. Better description of the charge behaviour will help to optimise liner design. This may minimize energy consumption and increase the efficiency of the milling process.

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